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Distance Antimagic Labeling of the Ladder Graph

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Abstract

Let G be a graph of order n. Let $f : V(G) \longrightarrow \{1, 2, \ldots, n\}$ be a bijection. The weight $w_f(v)$ of a vertex with respect to f is defined by $w_f(v) = \sum_{x \in N(v)} f(x)$. The labeling f is said to be distance antimagic if $w_f(u) \neq w_f(v)$ for every pair of vertices $u, v \in V(G)$. If the graph G admits such a labeling then G is said to be a distance antimagic graph. In this paper we investigate the existence of distance antimagic labeling in the ladder graph $L_n \cong P_2 \square P_n$.

Keywords: Distance antimagic graphs, antimagic labeling, arbitrarily distance antimagic.
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1 Introduction

By a graph $G = (V, E)$, we mean a finite undirected graph without loops, multiple edges or isolated vertices. For graph theoretic terminology we refer

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to West [7].

Most of the graph labeling methods trace their origin to the concept of β-valuation introduced by Rosa [6]. For a general overview of graph labeling we refer to the dynamic survey by Gallian [2].

Let G be a graph of order n. Let $f : V(G) \longrightarrow \{1, 2, \ldots, n\}$ be a bijection. The weight $w_f(v)$ of a vertex v is defined by $w_f(v) = \sum_{x \in N(v)} f(x)$ where $N(v)$ is the open neighbourhood of the vertex v. The labeling f is said to be distance antimagic if $w_f(u) \neq w_f(v)$ for every pair of vertices $u, v \in V(G)$. If the graph G admits such a labeling, then G is said to be a distance antimagic graph. Many classes of graphs are known to be distance antimagic. For details one may refer to Gallian [2] and Arumugam *et al.* [1]. In [5] Kamatchi and Arumugam posed the following problem:

Problem 1.1 *If G is a distance antimagic graph, is it true that $G + K_1$ and $G + K_2$ are distance antimagic?*

In [3] Handa *et al.* solved the problem in the affirmative. They introduced the concept of arbitrarily distance antimagic labeling as a tool to study distance antimagic labeling of join of two graphs.

Definition 1.2 A graph G of order n is said to be arbitrarily distance antimagic if there exists a bijection $f : V \longrightarrow \{1, 2, \ldots, n\}$ such that $w_{f_k}(u) \neq w_{f_k}(v)$ for any two distinct vertices u and v and for any $k \geq 0$. The labeling f with this property is called an arbitrarily distance antimagic labeling of G.

The following results are proved in [3].

Proposition 1.3 *[3] Any r-regular distance antimagic graph G is arbitrarily distance antimagic.*

Theorem 1.4 *[3] Let f be a distance antimagic labeling of a graph G of order n. If $w_f(u) < w_f(v)$ whenever $deg(u) < deg(v)$, then G is arbitrarily distance antimagic.*

Proposition 1.5 *[3] Let G_1 and G_2 be two graphs of order n_1 and n_2 with arbitrarily distance antimagic labelings f_1 and f_2 respectively,and let $n_1 \leq n_2$. Let $x \in V(G_1)$ be the vertex with lowest weight under f_1 and $y \in V(G_2)$ be the vertex with highest weight under f_2. If*

$$w_{f_1}(x) + \sum_{i=1}^{n_2} (n_1 + i) > w_{f_2}(y) + \Delta(G_2) n_1 + \sum_{i=1}^{n_1} i \tag{1}$$

then $G_1 + G_2$ is distance antimagic.

Since $n_1 \leq n_2$ the above inequality reduces to

$$w_{f_1}(x) + n_1 n_2 > w_{f_2}(y) + n_1 \Delta(G_2) \tag{2}$$

Theorem 1.6 *[3] Let G be a distance antimagic graph of order $n \geq 3$ with distance antimagic labeling f such that the highest weight under f is less than or equal to $\frac{n(n+1)}{2} - 3$. Then $G + K_3$ is distance antimagic.*

Handa *et al.* [4] obtained arbitrarily distance antimagic labeling for the graphs rP_n, generalized Petersen graph $P(n,k)$ for $n \geq 5$, Harary graph $H_{4,n}$ for $n \neq 6$ and the join of these graphs.

2 Main Results

In this section we shall obtain an arbitrarily distance antimagic labeling of the ladder graph $L_n \cong P_n \square P_2$.

Theorem 2.1 *The ladder L_n $n \geq 3$ is arbitrarily distance antimagic.*

Proof We consider the following two cases:

Case 1: n odd

Arbitrarily distance antimagic labeling for the graphs L_3 and L_7 are given in figures 1 and 2 respectively. For $n \geq 5$, $n \neq 7$ we define a labeling $f : V(L_n) \to \{1, 2, \ldots, 2n\}$ as follows:

$$f(u_i) = \begin{cases} 1 & \text{if } i = 1 \\ 6 & \text{if } i = 2 \\ 8 & \text{if } i = n-1 \\ 3 & \text{if } i = n \\ 4 + 2i & \text{if } i = 3, 4, \ldots, n-2 \end{cases}$$

$$f(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 5 & \text{if } i = 2 \\ 7 & \text{if } i = n-1 \\ 4 & \text{if } i = n \\ 2n + 5 - 2i & \text{if } i = 3, 4, \ldots, n-2 \end{cases}$$

The induced vertex weights are a follows:

$$w(u_i) = \begin{cases} 8i & \text{if } i = 1, 2 \\ 12 & \text{if } i = n \\ 2n + 17 & \text{if } i = 3 \\ 2n + 10 & \text{if } i = n - 1 \\ 2n + 15 & \text{if } i = n - 2 \\ 2n + 13 + 2i & \text{if } i = 4, 5, \ldots, n - 3 \end{cases}$$

$$w(v_i) = \begin{cases} 6 & \text{if } i = 1 \\ 21 & \text{if } i = n - 1 \\ 10 & \text{if } i = n \\ 2n + 7 & \text{if } i = 2 \\ 2n + 12 & \text{if } i = 3 \\ 4n + 14 - 2i & \text{if } i = 4, 5, \ldots, n - 2 \end{cases}$$

Case 2 : $n \geq 4$, n *is even.*

We define a labeling $f : V(L_n) \to \{1, 2, \ldots, 2n\}$ as follows:

$$f(u_i) = \begin{cases} 1 & \text{if } i = 1 \\ 6 & \text{if } i = 2 \\ 8 & \text{if } i = n - 1 \\ 3 & \text{if } i = n \\ 2n + 6 - 2i & \text{if } i = 3, 4, \ldots, n - 2 \end{cases}$$

$$f(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 5 & \text{if } i = 2 \\ 7 & \text{if } i = n - 1 \\ 4 & \text{if } i = n \\ 3 + 2i & \text{if } i = 3, 4, \ldots, n - 2 \end{cases}$$

The induced vertex weights are as follows:

$$w(u_i) = \begin{cases} 8 & \text{if } i = 1 \\ 2n+6 & \text{if } i = 2 \\ 20 & \text{if } i = n-1 \\ 12 & \text{if } i = n \\ 2n+13 & \text{if } i = 3 \\ 4n+15-2i & \text{if } i = 4, 5, \dots, n-2 \end{cases}$$

$$w(v_i) = \begin{cases} 6 & \text{if } i = 1 \\ 2n+11 & \text{if } i = n-1 \\ 10 & \text{if } i = n \\ 17 & \text{if } i = 2 \\ 2n+16 & \text{if } i = 3 \\ 2n+14 & \text{if } i = n-2 \\ 2n+12+2i & \text{if } i = 4, 5, \dots, n-3 \end{cases}$$

It is clear that all the weights are distinct. Furthermore the vertices with degree 2 receive the lowest weights in the labeling. Hence by Theorem 1.4 the labeling is arbitrarily distance antimagic. □

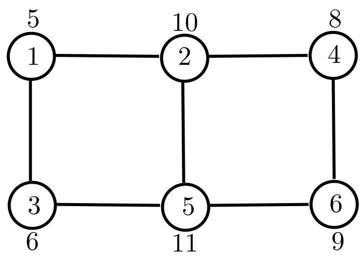

Figure 1. Arbitrarily distance antimagic labeling of L_3.

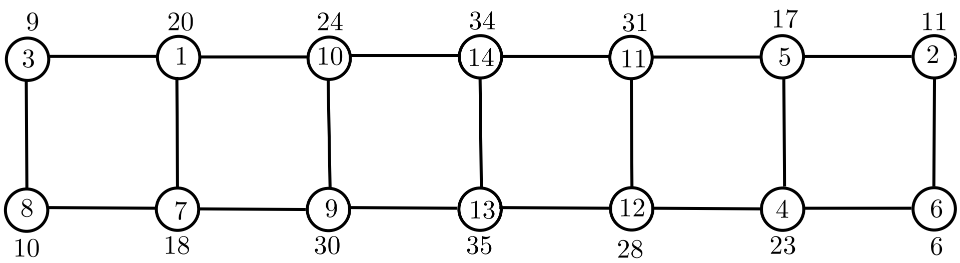

Figure 2. Arbitrarily distance antimagic labeling of L_7.

3 Conclusion

In this paper we have obtained an arbitrarily distance antimagic labeling for the graph $L_n \cong P_2 \square P_n$. However the question of whether or not the graph $P_n \square P_m$ is arbitrarily distance antimagic still remains open. We pose the general problem as follows:

Problem 3.1 *If the graphs G and H are distance antimagic, under what conditions is the graph $G \square H$ distance antimagic?*

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