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NUMERICAL SIMULATION OF FORCED CONVECTION IN OIL SANDS USING LATTICE BOLTZMANN METHOD

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ABSTRACT

Lattice Boltzmann method is used to simulate forced convection in oil sands (low permeable porous geometries). Fluid flows through a sandstone look alike square geometry with left wall of the geometry kept at higher temperature compared to other walls. Investigation is carried out to study influence of increased temperature on flow properties by observing the variation in velocity and temperature profiles for various permeability and porosity values, which were varied to match the geometrical properties of oil sands. Boundary conditions and the relaxation parameter are suitably defined to achieve convergence for low values of permeability. Simulation was carried out at low Reynolds number, which however, can be extended to higher values of Reynolds number.

Key words: Lattice Boltzmann Method, Porous Media, Forced Convection, Oil Sands

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1. INTRODUCTION

Use of elevated temperature to influence flow properties is well known in many fields that involve transport phenomena in porous media like petroleum extraction, underground water flow etc. Considering its wide applications, flow in porous media has been studied extensively over many decades. However, difficulties in practically replicating the real geometries and restrictions in numerical analysis have reserved a wide scope for improvements in studying flow in porous media. Nevertheless, the recent evolutions in computing technology and numerical tools have assisted in accomplishing this goal substantially. This has also helped in the evolution of Lattice Boltzmann method (LBM) as an effective numerical technique and has attracted a great focus for its ability to simulate flow in porous media. The very advantage of LBM is its simplicity in implementing complex boundary conditions. The construction of bounce back rules (i.e. boundary conditions) that governs the behavior of particles at the boundaries along with the relaxation parameter describes a specific problem. Thus, LBM is a much preferred numerical method to study flows in porous media compared to the conventional numerical techniques that involve solutions of Navier-Stokes equation.

A number of researchers have contributed in investigating forced convection in porous media. Seta et al. (2006) applied LBM to simulate forced convection in porous media using Brinkman-Forchheimer equation. The paper also justified that LBM can simulate natural convection in porous media in both Darcy and non-Darcy regions in an elementary volume scale. Guo and Zhao (2002) presented a LBM model for incompressible flows through porous media. Mehrizi et al. (2012) studied forced convection heat transfer in a square cavity with inlet and outlet slots and hot obstacle with and without porous medium. Grucelski and Pozorski (2013) applied single relaxation time variant LBM to the two–dimensional simulations of viscous fluids for circular cylinder and a computer-created porous medium. The research in this area is extensive. A large amount of this research has focused on flow through porous media for theoretical permeability values. Practically, since the permeability of real geometries like sand, oil sands etc. are very low, i.e. in the range of 10^{-10} to 10^{-6} [9], certain modification in the numerical methods are necessary. This will include applying suitable boundary conditions and the relaxation parameter. The present paper investigates forced convection in low permeable geometries like oil sands (sandstones), sand etc. which are seen to have permeability and porosity in the range of 10^{-6} to 10^{-10} and 0.4 to 0.6, respectively.

2. GOVERNING EQUATIONS AND NUMERICAL METHOD

2.1. Governing Equations

The generalized model for incompressible flow in porous media is given by Peng et al. (2003)

$$\nabla . u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + (u.\nabla)\left(\frac{u}{\varepsilon}\right) = -\frac{1}{\rho}\nabla(\varepsilon p) + v_e \nabla^2 u + F \tag{2}$$

where ρ is the fluid density, u and p are the volume-averaged velocity and pressure, respectively. F is the total body force given by

$$F = -\frac{\varepsilon v}{K}u - \frac{\varepsilon F_\varepsilon}{\sqrt{K}}|u|u + \varepsilon G \tag{3}$$

where v is the shear viscosity of the fluid and G is the body force. Permeability K and the Forcheimer's term F_ε are related to porosity ε as described by (Ergun 1952, Seta et al. 2006)

$$F_\varepsilon = \frac{1.75}{\sqrt{150\varepsilon^3}}, \quad K = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \tag{4}$$

where d_p is the solid particle diameter.

2.2. Lattice Boltzmann Method

The lattice Boltzmann method can be used to model hydrodynamic or mass transport phenomena by describing the particle distribution function $f_i(x,t)$ giving the probability that a fluid particle with velocity e_i enters the lattice site x at a time (Mohamad 2011, Chen and Doolen 1998). The subscript i represents the number of lattice links and $i=0$ corresponds to the particle at rest residing at the center. Present investigation is carried out for incompressible fluid flows and a nine-velocity model on a two-dimensional lattice (D2Q9). The distribution function $f_i(x,t)$ is governed by the lattice Boltzmann equation as (Guo and Zhao, 2002)

$$f_i(x+e_i dt, t+dt) - f_i(x,t) = \frac{f_i^{eq}(x,t) - f_i(x,t)}{\tau} + dtF_i \tag{5}$$

where dt is the time, τ is the relaxation time and is related to the kinematic viscosity υ by

$$\upsilon = c_s{}^2 dt\left(\tau - \frac{1}{2}\right) \tag{6}$$

c_s is the sound speed expressed by $c_s = dx/(\sqrt{3})dt = c/\sqrt{3}$ (c is the particle speed and dx is the lattice spacing). $f_i^{eq}(x,t)$ is the equilibrium distribution function for D2Q9 given by

$$f_i^{eq}(x,t) = w_i \rho(x,t)\left[1 + \frac{1}{c_s{}^2}(e_i.u(x,t)) + \frac{1}{2\varepsilon c_s{}^4}(e_i.u(x,t))^2 - \frac{u(x,t)^2}{2\varepsilon c_s^2}\right] \tag{7}$$

where $u(x,t)$ is the velocity and w_i is the weight coefficient.

The force term F_i is given by (Guo and Zhao, 2002)

$$F_i = w_i \rho\left(1 - \frac{1}{2\tau}\right)\left[\frac{e_i.F}{c_s^2} + \frac{uF:(e_i e_i - c_s^2 I}{\varepsilon c_s^2}\right] \tag{8}$$

The fluid velocity u is given by

$$u = \frac{u_t}{c_0 + \sqrt{c_0^2 + c_1|u_t|}} \tag{9}$$

where u_t is a temporal velocity given by

$$u_t = \sum_i e_i f_i + \frac{dt}{2}\rho\varepsilon G \tag{10}$$

The two parameters are given by

$$c_0 = \frac{1}{2}\left(1 + \varepsilon\frac{dt}{2}\frac{v}{K}\right), \quad c_1 = \varepsilon\frac{dt}{2}\frac{F_E}{\sqrt{K}} \tag{11}$$

2.3. Energy Equation

The thermal lattice BGK model proposed by Peng et al. (2003) is used to model heat transfer

$$g_i(x+e_i dt, t+dt) - g_i(x,t) = \frac{g_i^{eq}(x,t) - g_i(x,t)}{\tau_g} \quad (12)$$

$g_i(x,t)$ is the thermal distribution function, τ_g the relaxation time and $g_i^{eq}(x,t)$ is the equilibrium distribution function given by (Seta et al. 2006, Zou and He 1997)

$$g_0^{eq}(x,t) = -\frac{2\rho\varepsilon}{3}\frac{u^2}{c_s^2} \quad (13a)$$

$$g_i^{eq}(x,t) = \frac{\rho\varepsilon}{9}\left[\frac{3}{2} + \frac{3}{2}\frac{e_i.u}{c_s^2} + \frac{9}{2}\frac{(e_i.u)^2}{c_s^4} - \frac{3}{2}\frac{u^2}{c_s^2}\right](i = 1,2,3,4) \quad (13b)$$

$$g_i^{eq}(x,t) = \frac{\rho\varepsilon}{36}\left[3 + \frac{6}{2}\frac{e_i.u}{c_s^2} + \frac{9}{2}\frac{(e_i.u)^2}{c_s^4} - \frac{3}{2}\frac{u^2}{c_s^2}\right](i = 5,6,7,8) \quad (13c)$$

Reynolds number, $\mathrm{Re} = \frac{u_o L}{\upsilon}$, where u_o and L is the characteristic velocity and the characteristic length, respectively.

2.4. Boundary Conditions

Second order bounce back rule for non-equilibrium distribution function f_i is used to determine velocity on the four walls. The distribution functions are given by Zou and He (1997)

$$f_a^{neq} = f_b^{neq} \quad (14)$$

where b is the opposite direction of a .

For energy distribution function g_i, second order extrapolation rule is used on the left wall and the boundary conditions for all other walls were defined as per method introduced by D'Orazio et al (2004). The distribution functions on right wall were defined as

$$g_1 = \frac{6}{9}(1 - T_p),\ g_5 = \frac{1}{36}T^{eq}(1 + 3(u+v)),\ g_8 = \frac{1}{36}T^{eq}(1 + 3(u-v)) \quad (15a)$$

where $T_p = g_0 + g_2 + g_3 + g_4 + g_6 + g_7$ and $T^{eq} = \frac{6(1-T_p)}{1+3u}$. (15b)

The distribution functions on the top wall were defined as

$$g_4 = \frac{1}{9}T^{eq}(1+3u),\ g_7 = \frac{1}{36}T^{eq}(1 + 3(u+v)),\ g_8 = \frac{1}{36}T^{eq}(1 + 3(u-v)) \quad (16a)$$

where $T_p = g_0 + g_1 + g_2 + g_3 + g_5 + g_6$

and $T^{eq} = \frac{-6T_p}{1+3u}$. (16b)

The distribution functions on the bottom wall were defined as

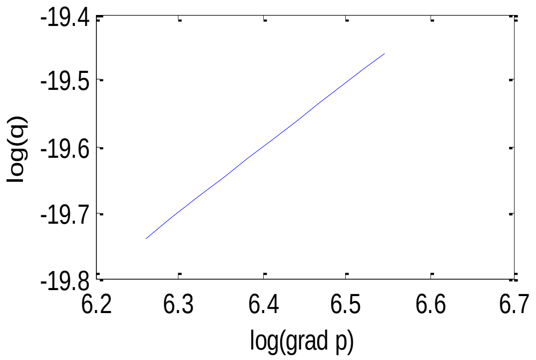
$$g_2 = \frac{1}{9}T^{eq}(1+3u),\ g_5 = \frac{1}{36}T^{eq}(1 + 3(u+v)),\ g_6 = \frac{1}{36}T^{eq}(1 + 3(u-v)) \quad (17a)$$

where $T_p = g_0 + g_1 + g_3 + g_4 + g_7 + g_9$

and $T^{eq} = \frac{-6T_p}{1+3u}$ (17b)

2.5. Problem Description and Numerical Implementation

Fluid flows through a square geometry containing porous media with the left wall kept at a normalized temperature and all other walls adiabatic. Fluid enters the geometry with uniform velocity u_o through the left wall along horizontal direction. Permeability and porosity varies between 10^{-10} to 10^{-6} and 0.4 to 0.5, respectively, to match the geometrical properties of oil sands. Reynolds number Re varies from 0.1 to 10 and Prandl number Pr is taken as 3.8 (of water). The relaxation parameter τ is has is taken as 1.5.



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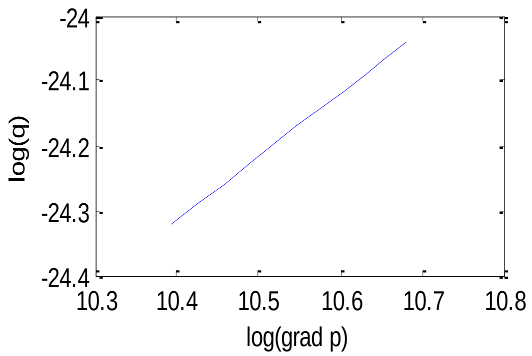

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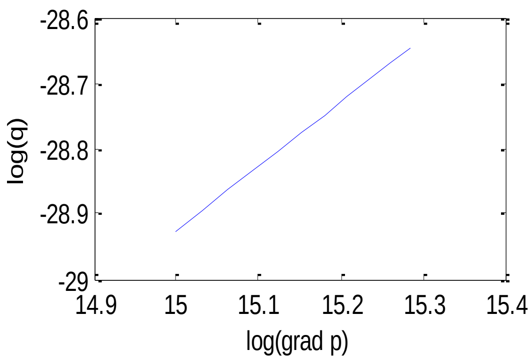


(c)

Figure 1 Plot of log (q) versus $\log(\nabla p)$ for Re=0.1 and $\varepsilon = 0.4$ for (a) $K = 10^{-6}$ (b) $K = 10^{-8}$ (c) $K = 10^{-10}$.

3. RESULTS AND DISCUSSION

The starting point for investigating fluid flow through porous media is the traditional modeling approach, typically, Darcy law, relating the pressure gradient ∇p to the pore fluid flux or Darcy flux q as

$$q = -\left(\frac{k}{\mu}\right)(\nabla p - \rho g) = -\left(\frac{k}{\mu}\right)\nabla \breve{p} \tag{18}$$

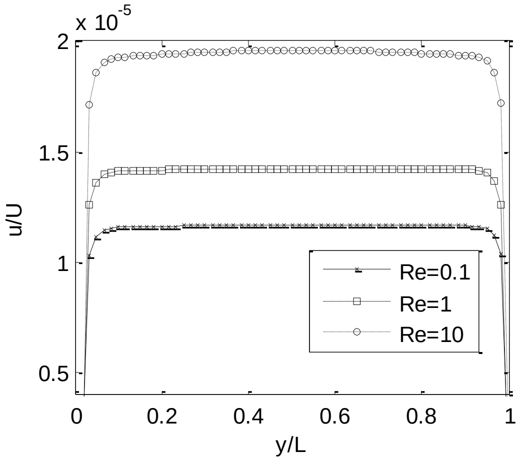
where q and p are defined in terms of u and ρ as $q = eu$ and $p = \rho/3$, respectively. Darcy law ensures a linear relationship between $\log(q)$ and $\log(\nabla p)$. Thus, simulation of fluid flow in porous media can be validated by showing that the plot of $\log(q)$ versus $\log(\nabla p)$ gives a straight line with gradient one.

While the paper carried out numerical investigation to study the influence of elevated temperature and the geometrical parameters on flow properties of fluid, the numerical strategy was verified by plotting $\log(q)$ and $\log(\nabla p)$ as shown in Fig 2. For the sake of space and invariability in values, only the plots for Re=0.1 are presented. Table 1 presents the gradient values of $\log(q)$ and $\log(\nabla p)$ and a fine agreement was observed between the results and the validating criteria of gradient one. Subsequently, simulation was carried out for governing parameters like Reynolds number, porosity, permeability etc. on the flow properties of fluid flowing through oil sands. This was achieved by varying Re between 0.1 to 10 keeping porosity and the permeability of the sandstone fixed, and then varying porosity and permeability to characterize the nature of the sandstone.

Fig 2 presents u-velocity profiles for $\varepsilon = 0.4$ at the centerline of the geometry as K varies from 10^{-10} to 10^{-6} at various Re. The u-velocity tends to become uniform along the geometry as flow becomes steady, as shown in Fig 2. The vertical component of flow is almost negligible at the order of 10^{-11}. As permeability of the sandstones is decreased, the velocity profiles follow the same trend, but with u-velocity decreased by same magnitude by which permeability is decreased. This implicates the strong influence of permeability flow properties of the fluid in sandstones. The magnitude of u-velocity is also influenced by Re, as seen in Fig 2, the velocity increases which is due to a stronger flow influenced by heat transfer through the fluid.

Table 1 Gradient values for $\log(q)$ and $\log(\nabla p)$

	$\varepsilon = 0.4$			$\varepsilon = 0.5$		
	Re			Re		
	0.1	1	10	0.1	1	10
$K = 10^{-6}$	0.9773	1.0011	0.9969	0.9764	1.0011	0.9994
$K = 10^{-8}$	0.9764	1.0011	0.9994	0.9764	1.0011	0.9994
$K = 10^{-10}$	0.9964	1.0011	0.9944	0.9764	1.0011	0.9994



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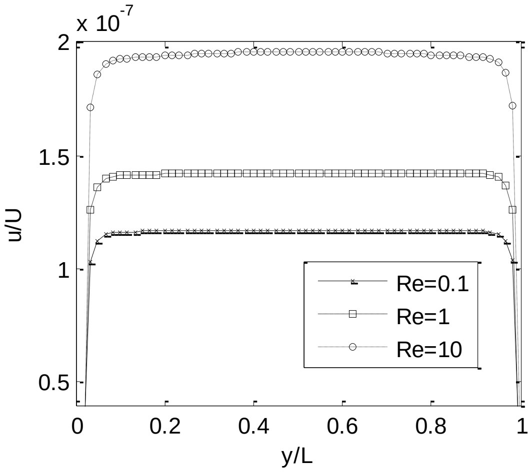

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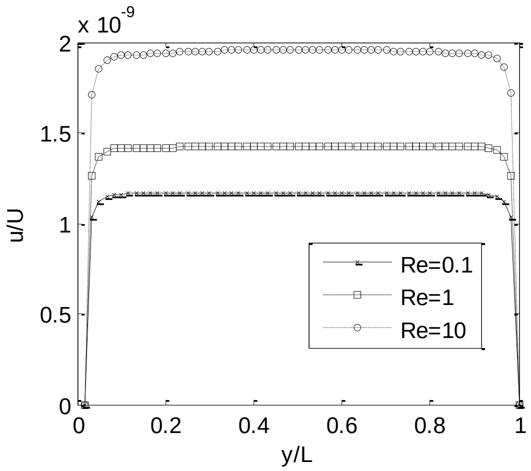
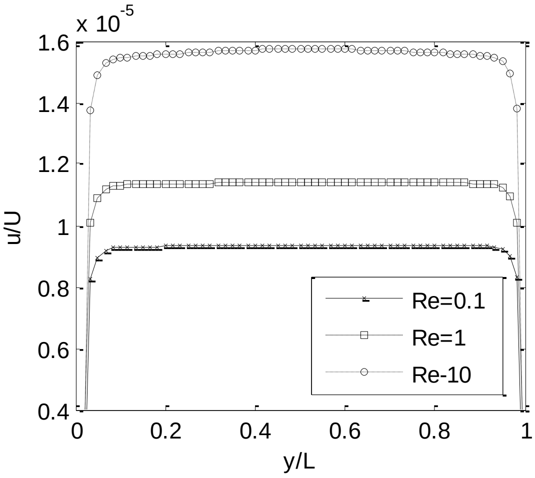


(c)

Figure 2 u-velocity profiles at the centerline for various values of Re at $\varepsilon = 0.4$ and (a) $K = 10^{-6}$ (b) $K = 10^{-8}$ (c) $K = 10^{-10}$.



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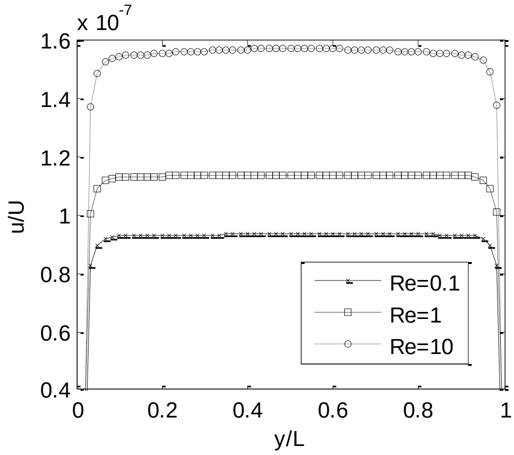

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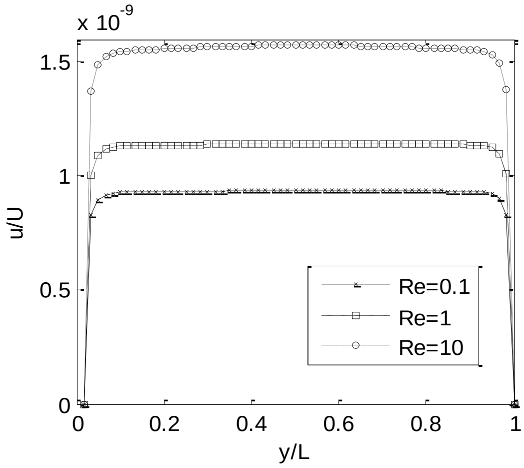
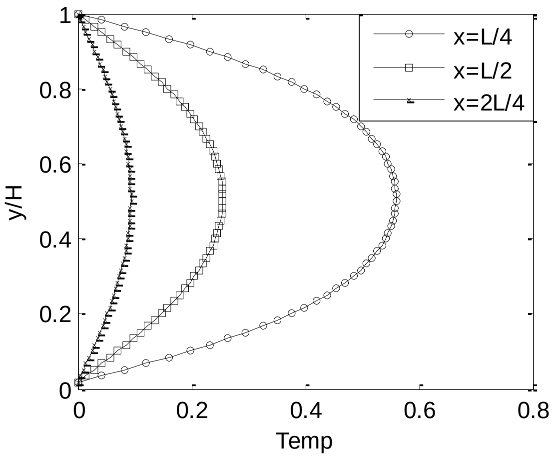


(c)

Figure 3 u-velocity profiles at the centerline for various values of Re at $\varepsilon = 0.5$ and (a) $K = 10^{-6}$ (b) $K = 10^{-8}$ (c) $K = 10^{-10}$.



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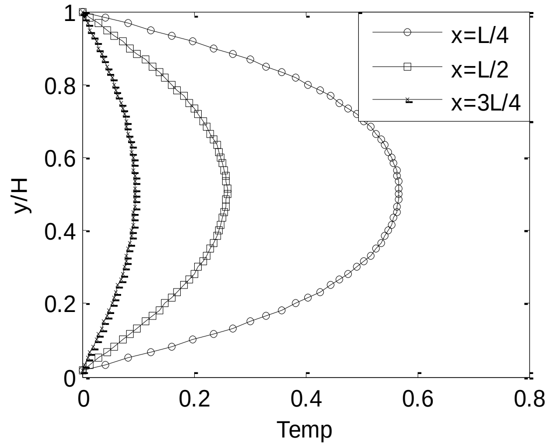

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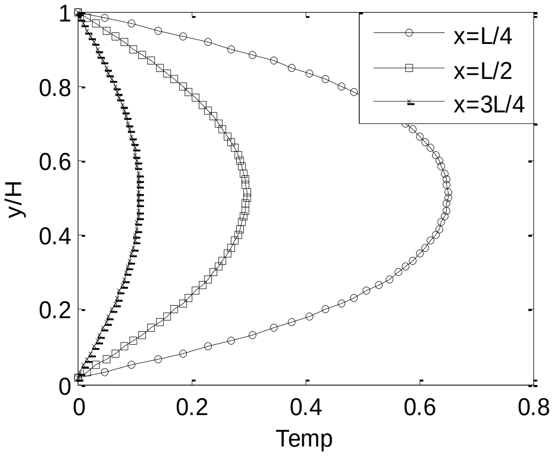
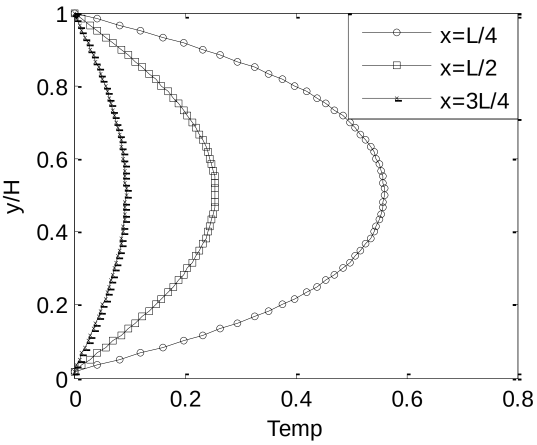


(c)

Figure 4 Temperature profiles at various cross section for $K = 10^{-6}$ and $\varepsilon = 0.4$ for (a) Re=0.1 (b) Re=1 (c) Re=10.



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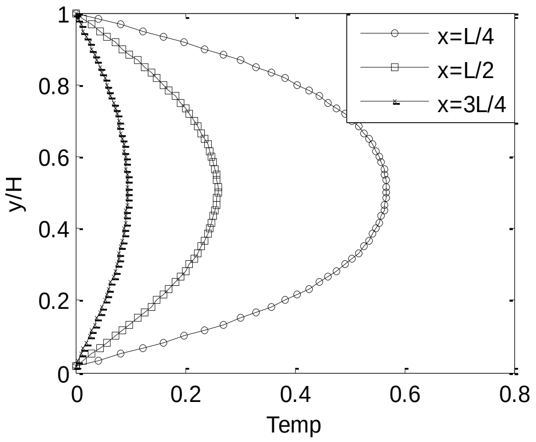

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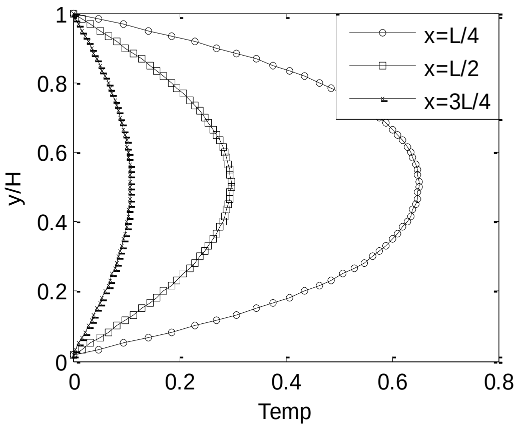

(c)

Figure 5 Temperature profiles at various cross section for $K = 10^{-6}$ and $\varepsilon = 0.5$ for (a) Re=0.1 (b) Re=1 (c) Re=10.

Since the permeability and Re are small, non-linear linear resistance is neglected, which become significant at higher values of K and Re. Fig 2-3 also demonstrates the influence of ε on flow properties. As ε is increased from 0.4 to 0.5, the u-velocity profiles decrease significantly, naturally, because the void space in the geometry increases resulting in a weaker flow through it. This agrees with the rheological behavior of the fluid when passing though a medium with bigger void portion. Since sand and oil sands are believed to have porosity not more than 0.5 (Bejan and Nield, 2013), further values of ε are not considered. The temperature profiles at different cross sections, as Re varies from 0.1 to 10, are presented for $K = 10^{-6}$ and $\varepsilon = 0.4$ in Fig 4. As Re increases from 0.1 to 10, the strength of heat transfer increases, indicating a significant increase in heat transfer though the fluid. Fig 5 presents the temperature profiles at the same cross section for $K = 10^{-6}$ and $\varepsilon = 0.5$. No significant influence of variation in K and ε is seen on the temperature profiles, and the plots follow a similar pattern as for $K = 10^{-6}$, thus, not included. This suggest that the variation in temperature profiles is only due to the variation in Re. The temperature profiles follow the same trend, both in magnitude and variation. These figures suggest that a better way to enhance flow through low permeable geometries can be achieved by elevating temperature to influence the flow strength.

4. CONCLUSION

Investigation was carried out to study the influence of elevated temperature and geometrical parameters on fluid flow in oil sands. LBM was validated as an appropriate numerical tool to simulate forced convection in porous media and applied to fluid flows in low permeable geometries. The results significantly depend on boundary conditions and the relaxation parameter, which had to be appropriately defined to simulate flows in such geometries. The results ascertain that elevated temperature plays an important role in influencing flow properties in low permeable geometries, which may be of significant application in petroleum extraction and groundwater flow etc. The geometric parameters of the porous media has very negligible influence on the flow properties compared to external influences, and a better way to influence the flow properties would be by influencing the strength of the flow by elevated temperature.

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