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3 authors:

adarsh Kumar Handa
BITS Pilani, K K Birla Goa

Aloysius Godinho
BITS Pilani, K K Birla Goa

Tarkeshwar Singh
BITS Pilani, K K Birla Goa

Some Distance Antimagic Labeled Graphs

Adarsh K. Handa*, Aloysius Godinho, and T. Singh

Birla Institute of Technology and Science Pilani,
K K Birla Goa Campus, Goa, India.
{p2013100,p2014001,tksingh}@goa.bits-pilani.ac.in

Abstract. Let G be a graph of order n. A bijection $f : V(G) \longrightarrow \{1, 2, \ldots, n\}$ is said to be distance antimagic if for every vertex v the vertex weight defined by $w_f(v) = \sum_{x \in N(v)} f(x)$ is distinct. The graph which admits such a labeling is called a distance antimagic graph. For a positive integer k, define $f_k : V(G) \longrightarrow \{1+k, 2+k, \ldots, n+k\}$ by $f_k(x) = f(x)+k$. If $w_{f_k}(u) \neq w_{f_k}(v)$ for every pair of vertices $u, v \in V$, for any $k \geq 0$ then f is said to be an *arbitrarily distance antimagic labeling* and the graph which admits such a labeling is said to be an *arbitrarily distance antimagic graph.* In this paper, we provide arbitrarily distance antimagic labelings for rP_n, generalised Petersen graph $P(n,k),\ n \geq 5$, Harary graph $H_{4,n}$ for $n \neq 6$ and also prove that join of these graphs is distance antimagic.

Keywords: Distance antimagic graphs, antimagic labeling.
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1 Introduction

By a graph we mean a finite undirected graph without loops and multiple edges. Throughout this paper, we consider simple graphs without isolates. For graph theoretic terminologies and notations we refer to West [7].

Graph labeling is an assignment of numbers to graph elements such as vertices or edges or both. The origin of graph labeling can be traced back to the concept of $\beta - valuations$ introduced by Rosa [6]. For a general overview of the current developments in graph labeling we refer to the dynamic survey by Gallian [3].

Let $G = (V, E)$ be a graph of order n. Let $f : V \rightarrow \{1, 2, \ldots, n\}$ be a bijection. For each vertex v, define the weight of v as $w_f(v) = \sum_{x \in N(v)} f(x)$. Then f is said to be a distance magic labeling of G if for every pair of vertices u and v, $w_f(u) = w_f(v)$ (cf.: $[1, 3, 5, 8]$) .

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* Also senior lecturer in mathematics at Padre Conceicao College of Engineering.

A natural question arises:*Is it possible to assign a bijection f to the vertices of the graph G such that $w_f(u) \neq w_f(v)$ for every pair of vertices $u, v \in V$?* A labeling which satisfies this condition is known as distance antimagic labeling and a graph which admits such a labeling is called a *distance antimagic graph.* This topic is studied extensively by Arumugam and Kamatchi [5].

Arumugam *et al.*[2] have proved that the path $P_n,\ n \neq 3$, the cycle $C_n,\ n \neq 4$, the wheel $W_n,\ n \neq 4$, and the graph $G = rK_2 + K_1$ are distance antimagic. They also posed the following problem:

Problem : If G is distance antimagic, are $G+K_1$, $G+K_2$ distance antimagic?

Handa *et al.* [4] have introduced the concept of arbitrarily distance antimagic labeling of a graph as follows:

Let $f : V(G) \to \{1, 2, \ldots, n\}$ be a distance antimagic labeling of a graph G. For a positive integer k, define $f_k : V(G) \longrightarrow \{1+k, 2+k, \ldots, n+k\}$ by $f_k(x) = f(x) + k$. If $w_{f_k}(u) \neq w_{f_k}(v)$ for every pair of vertices $u, v \in V$, for any $k \geq 0$ then f is called an *arbitrarily distance antimagic labeling* and the graph which admits such a labeling is said to be an *arbitrarily distance antimagic graph.* Note that an arbitrarily distance antimagic graph is always distance antimagic. But the converse is not true. Using the notion of arbitrarily distance antimagic labeling of graphs, they have answered the above problem in an affirmative way and have also proved that join of two graphs, in particular $P_n + P_m,\ P_n + C_m,\ P_n + W_m$ and $C_n + W_n$ are distance antimagic (cf.:[4]). In [4] they have posed the following problem:

Problem: If G and H are distance antimagic, is $G + H$ distance antimagic?

The following results are useful for our investigation.

Proposition 1. *[4] Any r-regular distance antimagic graph G is arbitrarily distance antimagic.*

Theorem 1. *[4] Let f be a distance antimagic labeling of a graph G of order n. If $w_f(u) < w_f(v)$ whenever $deg(u) < deg(v)$ then G is arbitrarily distance antimagic.*

Proposition 2. *[4] Let G_1 and G_2 be two graphs of order n_1 and n_2 with arbitrarily distance antimagic labelings f_1 and f_2 respectively, such that $n_1 \leq n_2$. Let $x \in V(G_1)$ be the vertex with lowest weight under f_1 and $y \in V(G_2)$ be the vertex with highest weight under f_2. If*

$$w_{f_1}(x) + \sum_{i=1}^{n_2}(n_1 + i) > w_{f_2}(y) + \Delta(G_2)n_1 + \sum_{i=1}^{n_1} i \tag{1}$$

then $G_1 + G_2$ is distance antimagic.

Since $n_1 \leq n_2$ the above inequality reduces to

$$w_{f_1}(x) + n_1 n_2 > w_{f_2}(y) + n_1 \Delta(G_2) \tag{2}$$

Theorem 2. *[4] Let G_1 and G_2 be graphs of order at least 4 which are arbitrarily distance antimagic and let $\Delta(G_1), \Delta(G_2) \leq 2$. Then $G_1 + G_2$ is distance antimagic.*

Theorem 3. *[4] Let G be a distance antimagic graph of order $n \geq 3$ with distance antimagic labeling f such that the highest weight under f is less than or equal to $\frac{n(n+1)}{2} - 3$. Then $G + K_3$ is distance antimagic.*

In this paper, we obtain arbitrarily distance antimagic labelings for the graphs rP_n, generalised Petersen graph $P(n,k)$ for $n \geq 5$, Harary graph $H_{4,n}$ for $n \neq 6$ and also prove that join of these graphs is distance antimagic.

2 Main Results

The graph rP_n is the disjoint union of r copies of P_n with vertex set $V = \{v_{i,j} \mid 1 \leq i \leq r,\ 1 \leq j \leq n\}$ where the sequence $v_{i,1}, v_{i,2}, \ldots, v_{i,n}$ (Figure 1) denotes the vertices of i^{th} path in rP_n.

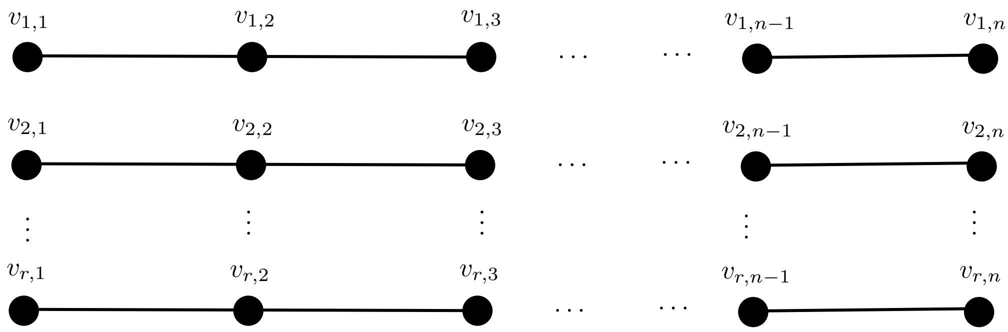

Fig. 1. Union of paths rP_n

Lemma 1. *For $r \in \mathbb{N}$ and odd $n \geq 5$, rP_n is arbitrarily distance antimagic.*

Proof. It is sufficient to provide a distance antimagic labeling of rP_n for odd $n \geq 5$. We define a labeling $f : V \to \{1, 2, \cdots, rn\}$ in each of the following cases:

Case 1 : $n \equiv 1 (\text{mod } 4)$.

$$f(v_{i,j}) = \begin{cases} 2i - 1 + r(j-2) & \text{if } 1 \leq i \leq r, \quad j = 2, 4, \ldots, \frac{n-1}{2}, \\ 2i + r(n-1-j) & \text{if } 1 \leq i \leq r, \quad j = \frac{n+3}{2}, \frac{n+3}{2} + 2, \ldots, n-1, \\ nr - \frac{(i-1)(n+1)}{2} - \frac{j-1}{2} & \text{if } 1 \leq i \leq r, \quad j = 1, 3, 5, \ldots, n. \end{cases}$$

then the vertex weights are as follows:

$$w_f(v_{i,j}) = \begin{cases} 2nr-(i-1)(n+1)-j+1 & \text{if } 1 \le i \le r, \quad j = 2,4,\dots,n-1, \\ 4i-2+r(2j-4) & \text{if } 1 \le i \le r, \quad j = 3,5,\dots,\frac{n-3}{2}, \\ 4i+2r(n-j-1) & \text{if } 1 \le i \le r, \quad \frac{n+5}{2},\frac{n+5}{2}+2,\dots,n-2, \\ 4i-1+r(n-5) & \text{if } 1 \le i \le r, \quad j = \frac{n+1}{2}, \\ 2i-1 & \text{if } 1 \le i \le r, \quad j = 1, \\ 2i & \text{if } 1 \le i \le r, \quad j = n. \end{cases}$$

Case 2 : $n \equiv 3 (\mathrm{mod}\ 4)$.

$$f(v_{i,j}) = \begin{cases} 2i-1+r(j-2) & \text{if } 1 \le i \le r, \quad j = 2,4,\dots,\frac{n-3}{2}, \\ 2i+r(n-1-j) & \text{if } 1 \le i \le r, \quad j = \frac{n+5}{2},\frac{n+5}{2}+2,\dots,n-1, \\ i+\frac{r(n-3)}{2} & \text{if } 1 \le i \le r, \quad j = \frac{n+1}{2}, \\ nr-\frac{(i-1)(n+1)}{2}-(\frac{j-1}{2}) & \text{if } 1 \le i \le r, \quad j = 1,3,5,\dots,n. \end{cases}$$

the vertex weights are as follows:

$$w_f(v_{i,j}) = \begin{cases} 2nr-(i-1)(n+1)-j+1 & \text{if } 1 \le i \le r, \quad j = 2,4,\dots,n-1, \\ 4i-2+r(2j-4) & \text{if } 1 \le i \le r, \quad j = 3,5,\dots,\frac{n-5}{2}, \\ 4i+2r(n-j-1) & \text{if } 1 \le i \le r, \quad \frac{n+7}{2},\frac{n+7}{2}+2,\dots,n-3, \\ 3i-1+r(n-5) & \text{if } 1 \le i \le r, \quad j = \frac{n-1}{2}, \\ 3i+r(n-5) & \text{if } 1 \le i \le r, \quad j = \frac{n+3}{2}, \\ 2i-1 & \text{if } 1 \le i \le r, \quad j = 1, \\ 2i & \text{if } 1 \le i \le r, \quad j = n. \end{cases}$$

In each of the above cases, it is easy to check that f is a bijection and the weights of the vertices are distinct. Since the lowest weights are assigned to the pendent vertices, it follows from Theorem 1 that the labeling f is an arbitrarily distance antimagic labeling of rP_n. □

Lemma 2. *The graph rP_n is arbitrarily distance antimagic for all even $n \ge 4$.*

Proof. It is sufficient to provide a distance antimagic labeling for rP_n for even $n \ge 4$. We define a distance antimagic labeling $f : V \to \{1, 2, \cdots, rn\}$ as follows:

$$f(v_{i,j}) = \begin{cases} 2i+r(j-2)-1 & \text{if } 1 \le i \le r, \quad j = 2,4,\dots,n, \\ 2i+r(n-1-j) & \text{if } 1 \le i \le r, \quad j = 1,3,\dots,n-1. \end{cases}$$

then the vertex weights are as follows:

$$w_f(v_{i,j}) = \begin{cases} 4i - 2 + r(2j-4) & \text{if } 1 \le i \le r, \quad j = 3,5,\ldots,n-1, \\ 4i + 2r(n-j-1) & \text{if } 1 \le i \le r, \quad 2,4\ldots,n-2, \\ 2i-1 & \text{if } 1 \le i \le r, \quad j = 1, \\ 2i & \text{if } 1 \le i \le r, \quad j = n. \end{cases}$$

It is easy to check that f is a bijection and the weights of the vertices are distinct. Since the lowest weights are assigned to the pendent vertices it follows from Theorem 1 that the labeling f is an arbitrarily distance antimagic labeling of rP_n. □

Lemma 3. rP_5 *is arbitrarily distance antimagic.*

Proof. It is sufficient to provide a distance antimagic labeling of rP_5. We define a labeling $f : V \to \{1, 2, \cdots, rn\}$ in each of the following cases:

Case 1 : r is even.

$$f(v_{i,j}) = \begin{cases} 3r - i & \text{if } j = 4, 1 \le i \le r, \\ 2i-1 & \text{if } j = 2,\ 1 \le i \le r, \\ 4i-2 & \text{if } j = 1,\ 1 \le i \le \frac{r}{2} - 1, \\ 4i & \text{if } j = 5,\ 1 \le i \le \frac{r}{2} - 1, \\ 4r-1-i & \text{if } j = 3,\ 1 \le i \le \frac{r}{2} - 1, \\ 2r-2 & \text{if } i = r\ j = 5. \end{cases}$$

Let S be the set of vertex labels assigned above, therefore $S = \{3r - 1, 3r-2, \ldots, 2r\} \cup \{1, 3, 5, \ldots, 2r-1\} \cup \{2, 6, \ldots, 2r-6\} \cup \{4, 8, \ldots, 2r-4\} \cup \{4r-2, 4r-3, \ldots, \frac{7r}{2}\} \cup \{2r-2\}$. Let $A = \{1, 2, 3, \ldots, 5r\} \backslash S$. Therefore $A = \{3r, 3r+1, \ldots, \frac{7r-2}{2}\} \cup \{4r-1, 4r, \ldots, 5r\}$. The number of elements in A is $\frac{3r}{2} + 2$. Now, we label the remaining vertices of rP_5 with the labels in A from left to right whilst moving downwards. It is clear that label of all vertices are distinct.

Then the weight of vertices are as follows:

$$w_f(v_{i,j}) = \begin{cases} 2i-1 & \text{if } j = 1,\ 1 \le i \le r, \\ 3r-i & \text{if } j = 5,\ 1 \le i \le r, \\ 3r+i-1 & \text{if } j = 3,\ 1 \le i \le r, \\ 4r+3(i-1) & \text{if } j = 2,\ 1 \le i \le \frac{r}{2} - 1, \\ 4r+3i-1 & \text{if } j = 4,\ 1 \le i \le \frac{r}{2} - 1, \\ 7r-2 & \text{if } i = r, j = 4. \end{cases}$$

The weights of the remaining vertices which are labeled from the set A are in increasing order, the weights of the vertices increase as we move downward from left to right. Hence they are distinct.

Case 2 : r is odd.

$$f(v_{i,j}) = \begin{cases} 3r-i & \text{if } j=4, 1 \le i \le r, \\ 2i-1 & \text{if } j=2,\ 1 \le i \le r, \\ 4i-2 & \text{if } j=1,\ 1 \le i \le \lfloor \frac{r}{2} \rfloor, \\ 4i & \text{if } j=5,\ 1 \le i \le \lfloor \frac{r}{2} \rfloor, \\ 4r-1-i & \text{if } \ j=3,\ 1 \le i \le \lfloor \frac{r}{2} \rfloor. \end{cases}$$

Let S be the set of vertex labels assigned above, therefore $S = \{3r-1, 3r-2, \dots, 2r\} \cup \{1,3,5,\dots,2r-1\} \cup \{2,6,\dots,4\lfloor \frac{r}{2} \rfloor - 2\} \cup \{4,8,\dots,4\lfloor \frac{r}{2} \rfloor\} \cup \{4r-2, 4r-3, \dots, 4r-1-\lfloor \frac{r}{2} \rfloor\}$. Let $A = \{1,2,3,\dots,5r\} \backslash S$. Therefore $A = \{3r, 3r+1, \dots, \lfloor \frac{7r-2}{2} \rfloor\} \cup \{4r-1, 4r, \dots, 5r\}$. Now, we label the remaining vertices of rP_5 with the numbers from A from left to right whilst moving downwards (refer Figure 1). It is clear that the labels of all the vertices are distinct.

The weight of vertices are as follows:

$$w_f(v_{i,j}) = \begin{cases} 2i-1 & \text{if } j=1,\ 1 \le i \le r, \\ 3r-i & \text{if } j=5,\ 1 \le i \le r, \\ 3r+i-1 & \text{if } j=3,\ 1 \le i \le r, \\ 4r+3(i-1) & \text{if } j=2,\ 1 \le i \le \frac{r}{2}, \\ 4r+3i-1 & \text{if } j=4,\ 1 \le i \le \frac{r}{2}. \end{cases}$$

The weights of the remaining vertices which are labeled from the set A are in increasing order, therefore the weights of the vertices increase as we move downward from left to right. Hence they are distinct.

In each of the above cases, since the lowest weights are assigned to the pendent vertices, it follows from Theorem 1 that the labeling f is an arbitrarily distance antimagic labeling. □

Observation 4. *Let* $V(rP_3) = \{v_{i,j} : 1 \le i \le r,\ 1 \le j \le 3\}$. *If* $f : V(rP_3) \to \{1,2,\dots,3r\}$ *is a bijection, then for any* i, $w_f(v_{i,1}) = w_f(v_{i,3})$. *Hence for any* $r \in \mathbb{N}$, *the graph* rP_3 *is not distance antimagic.*

Observation 5. *If* $f : V(rP_2) \to \{1,2,\dots,2r\}$ *be any bijection, then* $w_f(v_{i,1}) = f(v_{i,2})$ *and* $w_f(v_{i,2}) = f(v_{i,1})$. *Since the labels of all the vertices are distinct, it follows that all the vertex weights are also distinct. Therefore for any* $r \in \mathbb{N}$, *the graph* rP_2 *is arbitrarily distance antimagic.*

Theorem 6. *For* $r \in \mathbb{N}$, *the graph* rP_n *is arbitrarily distance antimagic if and only if* $n \neq 3$.

Proof. The proof of the theorem follows from Lemmas 1, 2, 3 and Observations 4, 5. □

Theorem 7. *Harary graph $H_{4,n}$ is arbitrarily distance antimagic for all $n \neq 6$.*

Proof. Let the vertex set of $H_{4,n}$ be $\{v_1, v_2, \ldots, v_n\}$. It is sufficient to provide a distance antimagic labeling of $H_{4,n}$. We define a labeling $f : V \to \{1, 2, \cdots, n\}$ in each of the following cases:

Case 1 : n is odd.
$f(v_i) = i, \; i = 1, 2, \ldots, n$.

The vertex weights are as follows:

$$w_f(v_i) = \begin{cases} 2n+4 & \text{if } i = 1, \\ 2n & \text{if } i = n, \\ 4i & \text{if } i = 3, 4, 5, \ldots, n-2, \\ 3n-4 & \text{if } i = n-1, \\ n+8 & \text{if } i = 2. \end{cases}$$

Case 2 : n is even. In this case we have the following two sub-cases:

Sub-case 1: $n \equiv 0 (\mathrm{mod} 4)$.

$$f(v_i) = \begin{cases} i & \text{if } i = 1, 2, \ldots, n-2, \\ n & \text{if } i = n-1, \\ n-1 & \text{if } i = n. \end{cases}$$

The vertex weights are as follows:

$$w_f(v_i) = \begin{cases} 2n+4 & \text{if } i = 1, \\ n+7 & \text{if } i = 2, \\ 4i & \text{if } i = 3, 4, 5, \ldots, n-4, \\ 4n-11 & \text{if } i = n-3, \\ 4n-8 & \text{if } i = n-2, \\ 3n-5 & \text{if } i = n-1, \\ 2n+1 & \text{if } i = n. \end{cases}$$

Sub-case 2: $n \equiv 2 (\mathrm{mod} 4)$.

$$f(v_i) = \begin{cases} i & \text{if } i = 1, 2, \ldots, n-3, \\ n & \text{if } i = n-2, \\ n-2 & \text{if } i = n-1, \\ n-1 & \text{if } i = n. \end{cases}$$

The vertex weights are as follows:

$$w_f(v_i) = \begin{cases} 2n+2 & \text{if } i = 1, \\ n+7 & \text{if } i = 2, \\ 4i & \text{if } i = \ 3, 4, 5, \ldots, n-5, \\ 4n-14 & \text{if } i = n-4, \\ 4n-11 & \text{if } i = n-3, \\ 4n-10 & \text{if } i = n-2, \\ 3n-3 & \text{if } i = n-1, \\ 2n+1 & \text{if } i = n. \end{cases}$$

In each of the above cases, it is easy to see that the labels of the vertices and weights of the vertices are also distinct. By Proposition 1 the labeling f is an arbitrarily distance antimagic labeling.

□

The illustration of the distance antimagic labeling for $H_{4,8}$ is shown in figure 2.

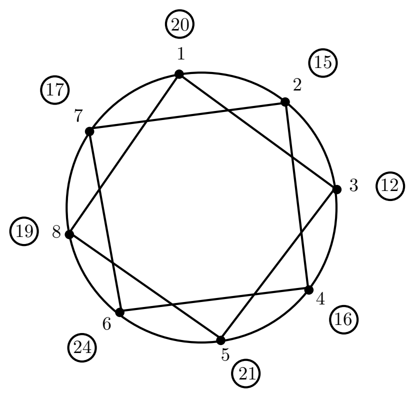

Fig. 2. Distance antimagic labeling of $H_{4,8}$

For $n \geq 5$ and $k < \frac{n}{2}$, the generalized Petersen graph $P(n,k)$ is a graph with vertex set $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and edge set $\{u_iu_{i+1}, u_iv_i, v_iv_{i+k} \ : \ 1 \leq i \leq n\}$ where the subscripts are taken modulo n.

Theorem 8. *The graph $P(n,k)$ is arbitrarily distance antimagic.*

Proof. It is sufficient to provide a distance antimagic labeling of $P(n, k)$. We define a lebeling $f : V \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(u_i) = 2i - 1, \quad 1 \leq i \leq n,$$

$$f(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 2(n - i + 2) & \text{if } 2 \leq i \leq n. \end{cases}$$

The weights of vertices are as follows:

$$w_f(u_i) = \begin{cases} 2(n + i + 1) & \text{if } 1 \leq i \leq n - 1, \\ 2n + 2 & \text{if } i = n \end{cases}$$

$$w_f(v_i) = \begin{cases} 2n - 2i + 7 & \text{if } 1 \leq i \leq k + 1, \\ 4n - 2i + 7 & \text{if } k + 2 \leq i \leq n - k + 1, \\ 6n - 2i + 7 & \text{if } n - k + 2 \leq i \leq n, \ k \geq 2. \end{cases}$$

It is easy to see that the labels of the vertices and the weights of the vertices are distinct. By Proposition 1 the labeling f is an arbitrarily distance antimagic labeling. □

3 Join of Graphs

Theorem 9. *For $r, k \in \mathbb{N}$, $n, m \geq 4$, graph $rP_n + kP_m$ is distance antimagic.*

Proof. The proof follows from Theorem 6 and Theorem 2. □

Theorem 10. *The graphs $rP_n + K_1$, $rP_n + K_2$ and $rP_n + K_3$ are distance antimagic for $n \geq 4$ and $n \neq 5$.*

Proof. The proof follows from Theorem 6 and Theorem 3. □

Theorem 10 also holds for $n = 2$. Thus we have the following corollary.

Corollary 1. *[5] The graph $G = rK_2 + K_1$ is distance antimagic.*

Theorem 11. *The graph $H_{4,n_1} + H_{4,n_2}$ is distance antimagic.*

Proof. From Theorem 7, the highest vertex weight is $\leq 4n - 8$ and the lowest vertex weight attained is at least 8. We have,

$$8 + \frac{n_2(n_2 + 1)}{2} + n_1 n_2 > 4n_2 - 8 + \frac{n_1(n_1 + 1)}{2} + 4n_1$$

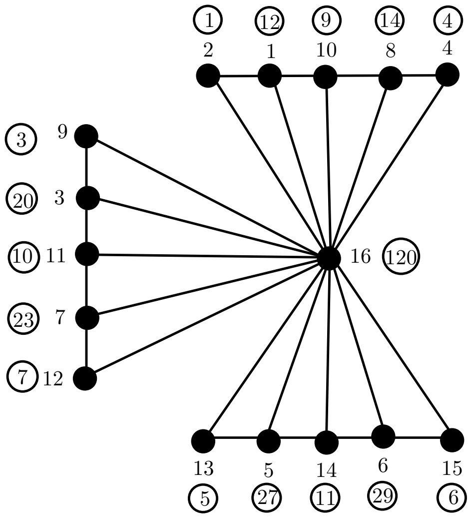

Fig. 3. Distance antimagic labeling of $3P_5 + K_1$

$$\Rightarrow 16 + (\frac{n_2(n_2+1)}{2} - \frac{n_1(n_1+1)}{2}) + n_1n_2 - 4n_2 - 4n_1 > 0$$

Thus the result follows. □

Theorem 12. *The graph $P(n,k) + H_{4,m}$, with $m, n \geq 5,\ m \neq 6$, is distance antimagic.*

Proof. By Theorem 7 and Theorem 8, $H_{4,m}$ and $P(n,k)$ are arbitrarily distance antimagic. Let f_1 and f_2 be the arbitrarily distance antimagic labeling of $H_{4,m}$ and $P(n,k)$ respectively. The highest vertex weight under f_1 for $H(4,m)$ is at most $4m - 10$ and the lowest weight is at least 8. The highest weight in $P(n,k)$ is at most $5n + 3$ and the lowest weight is at least 12. We have the following two cases:

Case 1: $2n \leq m$

From inequality (2) we have

$$12 + \frac{m(m+1)}{2} + 2mn > 4m - 10 + \frac{2n(2n+1)}{2} + 8n$$

$$\Rightarrow 2mn - 4m - 8n + 22 > 0$$

$$\Rightarrow m(2n-4) - 8n + 22 > 0$$

as $n \geq 5$.

Case 2 : $2n > m$.
From inequality (2) we have

$$8 + \frac{2n(2n+1)}{2} + 2mn > 5n + 3 + \frac{m(m+1)}{2} + 3m$$
$$\Rightarrow 2mn - 5n - 3m + 5 > 0$$
$$\Rightarrow 2n(m - \frac{5}{2}) - 3m + 5 > 0.$$

Since $n \geq 3$ and $m \geq 5$, this equality holds. Hence by Proposition 2, $P(n,k) + H_{4,m}$ is distance antimagic. This completes the proof. □

4 Conclusion and scope

In this paper, we have obtained arbitrarily distance antimagic labeling for the graphs rP_n, generalised Petersen graph $P(n,k)$ for $n \geq 5$, Harary graph $H_{4,n}$ for $n \neq 6$ and have also proved that the join of these graphs is distance antimagic. The following problems are remain open:

Problem 1: Characterize graphs which are distance antimagic.
Problem 2: If G and H are distance antimagic, is $G + H$ distance antimagic?

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